Towards Coinductive Theory Exploration

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(joint work with Yue Li, Henning Basold, John Power et al.)

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Outline

Problem statement

Solution

Technical details

FOL, Coinductively

- Any theory expressed in FOL may be seen inductively or coinductively,
- Depending on the chosen semantics

For example...

Given a theory in Horn Clause syntax:

$$
\begin{aligned}
G &::= \top \mid A \mid G \wedge G \mid G \vee G \mid \exists Var\ G \\
D &::= A \mid G \supset D \mid D \wedge D\ \mid \forall Var\ D
\end{aligned}
$$

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Coinductive models of theories in Horn Clause logic

- Take the sets of (finite and/or infinite) ground terms
- The coinductive model is the largest set of such terms such that it satisfies the given theory
- (The Inductive model is the smallest such set)
- Usually these models are given by fixed point (Knaster-Tarski) construction

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- (The Inductive model is the smallest such set)
- Usually these models are given by fixed point (Knaster-Tarski) construction

	least fixed point	greatest fixed point
finite terms	Least Herbrand models	Greatest Herbrand models
finite and infinite terms	Least Complete Herbrand models	Greatest Complete Herbrand models

Example 1

- Both inductive and coinductive semantics may suit

Example

$\kappa_1 : \forall x, nat\ x \supset nat\ (s\ x)$

$\kappa_2 : nat\ 0$

	least fixed point	greatest fixed point
finite terms	$\{nat\ 0, nat(s\ 0), \ldots\}$	$\{nat\ 0, nat(s\ 0), \ldots\}$
finite and infinite terms	$\{nat\ 0, nat(s\ 0), \ldots\}$	$\{nat\ 0, nat(s\ 0), \ldots, s^{\omega}\}$

Example 2

- ... only coinductive semantics may suit

Example

$\kappa_1 : \forall x, streamZ\ x \supset streamZ\ (scons\ 0\ x)$

	least fixed point	greatest fixed point
finite terms	$\emptyset$	$\emptyset$
finite and infinite terms	$\emptyset$	$\{streamZ(scons\ 0(scons\ 0\ \ldots))\}$

Example 3

- Either semantics may work well for certain fragment of the theory

Example

$\kappa_1 : \forall x, nat\ x \supset nat\ (s\ x)$

$\kappa_2 : nat\ 0$

$\kappa_3 : \forall x, nat\ x \wedge streamN\ y \supset streamN\ (scons\ x\ y)$

	least fixed point	greatest fixed point
finite terms	$\{nat\,0, nat(s\,0), \ldots\}$	$\{nat\ 0, nat(s\ 0), \ldots\}$
finite and infinite terms	$\{nat\,0, nat(s\,0), \ldots\}$	$\{nat\ 0, nat(s\ 0), \ldots, s^{\omega},$ $streamN(scons\ 0(scons\ 0\ \ldots)),$ $streamN(scons\ 0(scons\ 1\ \ldots)),$ $streamN(scons\ 1(scons\ 0\ \ldots)),$ $streamN(scons\ 1(scons\ 1\ \ldots)),$ $\ldots\}$

Operational semantics:

- State of the art is automated invariant discovery by means of loop detection

Example

$\forall x, \mathit{streamZ}\ x \supset \mathit{streamZ}\ (\mathit{scons}\ 0\ x)$

Resolution-based search:
$\mathit{streamZ}\ x \leadsto^{x/(\mathit{scons}\ 0\ x')} \mathit{streamZ}\ x' \leadsto$

- Terminate the loop with $x = (\mathit{scons}\ 0\ x)$.
- It is the coinductive invariant.

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- It is the coinductive invariant.

Recall:

	least fixed point	greatest fixed point
finite terms	$\emptyset$	$\emptyset$
finite and infinite terms	$\emptyset$	$\{streamZ(scons\ 0(scons\ 0\ \ldots))\}$

However, loop detection

- ... is not a very satisfactory solution

Why unsatisfactory?

1 it fails too often

Example

$\forall x, \mathit{from}\ (s\ x)\ y \supset \mathit{from}\ x\ (\mathit{scons}\ x\ y)$

Resolution-based search:
$\mathit{from}\ 0\ x \leadsto^{x/(\mathit{scons}\ 0\ x')} \mathit{from}\ (s\ 0)\ x' \leadsto$

- No unifier exists,
- loop detection fails to find coinductive invariant

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	least fixed point	greatest fixed point
finite terms	$\emptyset$	$\emptyset$
finite and infinite terms	$\emptyset$	$\{\mathit{from}\ 0(\mathit{scons}\ 0(\mathit{scons}\ (s0)\ \ldots))\}$

Why unsatisfactory?

2 it is a bad indicator for coinductive meaning of the theory

(Works well with existential, but not universal coinductive models)

Universal Coinductive Models

Example

$\kappa_1 : \forall x, p(f\ x) \supset p\ x$

	least fixed point	greatest fixed point
finite terms	$\emptyset$	$\{p\ a, p(f\ a), p(f\ f\ a), \ldots\}$
finite and infinite terms	$\emptyset$	$\{p\ a, p(f\ a), p(f\ f\ a), \ldots\ p\ f^{\omega}\ \}$

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Invariant search:

$p\ x \rightarrow p(f\ x) \rightarrow p(f\ f\ x) \rightarrow \ldots$

- The answer is: $x = f\ x$.
- However, f^{ω} is not all that there is in the model!

Universal Coinductive Models

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- The answer is: $x = f\ x$.
- However, f^{ω} is not all that there is in the model!

$p\ a \rightarrow p(f\ a) \rightarrow p(f\ f\ a) \rightarrow \ldots$

- fails to find a loop

Outline

Solution?

- Recast the problem of invariant search as a problem of coinductive theory exploration

Example 1

Example

$\forall x, streamZ\ x \supset streamZ\ (scons\ 0\ x)$

Resolution-based search:
$streamZ\ x \leadsto^{x/(scons\ 0\ x')} streamZ\ x' \leadsto$

- Terminate the loop with $x = (scons\ 0\ x)$.
- It is the coinductive invariant.
- Find and prove $streamZ(zstream)$
- for $zstream = fix\lambda\ x.scons\ 0\ x$

Example 2

Example

$\kappa_1 : \forall x, p(f \ x) \supset p \ x$

$p \ a \rightarrow p(f \ a) \rightarrow \ldots$

- ~~fails to find a loop~~
- Find and prove $\forall \ x, \ p \ x$
- Get $p \ a$ as a corollary

Outline

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Uniform proofs [Miller et al.]

- give proof-theoretic interpretation to goal-oriented proof search
- Uniform: – one rule applies at every stage of the proof
- Proven to be a fragment of intuitionistic logic

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- Uniform: – one rule applies at every stage of the proof
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FOHH and HOHH

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$$

Logical rules

$$\frac{}{\Sigma; P \longrightarrow \top} \; \top R$$

$$\frac{\Sigma; P \longrightarrow G_1 \quad \Sigma; P \longrightarrow G_2}{\Sigma; P \longrightarrow G_1 \wedge G_2} \; \wedge R$$

$$\frac{\Sigma; P \longrightarrow G_1}{\Sigma; P \longrightarrow G_1 \vee G_2} \; \vee R$$

$$\frac{\Sigma; P \longrightarrow G_2}{\Sigma; P \longrightarrow G_1 \vee G_2} \; \vee R$$

$$\frac{\Sigma; P, D \longrightarrow G}{\Sigma; P \longrightarrow D \supset G} \; \supset R$$

$$\frac{c : \tau, \Sigma; P \longrightarrow G\,[x := c]}{\Sigma; P \longrightarrow \forall_\tau x \;\, G} \; \forall R$$

$$\frac{\Sigma; P \longrightarrow G\,[x := N]}{\Sigma; P \longrightarrow \exists_\tau x \;\, G} \; \exists R$$

Backchaining (resolution) rules

$$\vdots$$

$$\frac{\Sigma; P \xrightarrow{D} A}{\Sigma; P \longrightarrow A}\ \textsc{Decide}$$

$$\frac{\Sigma; P \xrightarrow{D} A \quad \Sigma; P \longrightarrow G}{\Sigma; P \xrightarrow{G \supset D} A}\ \supset L$$

$$\frac{\Sigma; P \xrightarrow{D[x:=N]} A \quad \Sigma, \emptyset \vdash N : \tau}{\Sigma; P \xrightarrow{\forall_\tau x\ D} A}\ \forall L$$

COFIX rule for uniform proofs

$$\frac{\Sigma; P, M \longrightarrow \langle M \rangle}{\Sigma; P \looparrowright M} \ \text{COFIX}$$

COFIX rule for uniform proofs

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the guarding modality $\langle M \rangle$ must be discharged to get M
(this can be done if $\langle M \rangle$ is resolved ($=$ pattern matched) against a clause in P).
The successful proof ends with $\Sigma; P, M \longrightarrow M$.

Lucky case: trivial coinductive invariant

Example

$\kappa_1 : \forall x, p\ x \supset p\ x$
Find invariant for: $\underline{p\ a} \longrightarrow p\ a \longrightarrow \ldots ?$

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	least fixed point	greatest fixed point
finite terms	$\emptyset$	$\{p\ a\}$
finite and infinite terms	$\emptyset$	$\{p\ a\}$

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Find invariant for: $\underline{p\ a} \longrightarrow p\ a \longrightarrow \ldots ?$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P; p\ a \xrightarrow{p\ a} p\ a}\ \text{Initial} \qquad \dfrac{}{P; p\ a \xrightarrow{p\ a} p\ a}\ \text{Initial}}{P; p\ a \xrightarrow{p\ a \supset p\ a} \langle p\ a \rangle}\ \supset L}{P; p\ a \xrightarrow{\forall x, p\ x \supset p\ x} \langle p\ a \rangle}\ \forall L}{P; p\ a \longrightarrow \langle p\ a \rangle}\ \mathit{DECIDE}}{P \looparrowright p\ a}\ \text{COFIX}$$

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Find invariant for: $\underline{p\ a} \longrightarrow p\ a \longrightarrow \ldots$?

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QUIZ: which logic does this coinductive hypothesis and prove live in?

Not so lucky case: universal coinductive invariant

Example

$\kappa_1 : \forall x, p(f\ x) \supset p\ x$
Find invariant for: $\underline{p(a)} \longrightarrow p(f\ a) \longrightarrow p(f\ f\ a) \longrightarrow \ldots$?

Not so lucky case: universal coinductive invariant

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Example

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Find invariant for: $\underline{p(a)} \longrightarrow p(f\ a) \longrightarrow p(f\ f\ a) \longrightarrow \ldots ?$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P; p\ a \xrightarrow{p\ a} p(a)}\ \text{Initial} \qquad \dfrac{}{P; p\ a \longrightarrow p(f\ a)}\ ???}{P; p\ a \xrightarrow{p(f\ a) \supset p\ a} \langle p\ a \rangle}\ \supset L}{P; p\ a \xrightarrow{\forall x, p(f\ x) \supset p\ x} \langle p\ a \rangle}\ \forall L}{P; p\ a \longrightarrow \langle p\ a \rangle}\ DECIDE}{P \looparrowright p\ a}\ \text{COFIX}$$

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$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{}{P; \forall x, p\ x \xrightarrow{p\ a} p\ a}\ \text{Initial}
        \qquad
        \dfrac{
          \dfrac{
            \dfrac{}{P; \forall x, p\ x \xrightarrow{p\ (f\ a)} p\ (f\ a)}\ \text{Initial}
          }{P; \forall x, p\ x \xrightarrow{\forall x, p\ x} p\ (f\ a)}\ \forall L
        }{P; \forall x, p\ x \longrightarrow p\ (f\ a)}\ \text{DECIDE}
      }{P; \forall x, p\ x \xrightarrow{p\ (f\ a) \supset p\ a} \langle p\ a \rangle}\ \supset L
    }{P; \forall x, p\ x \xrightarrow{\forall x, p(f(x)) \supset p\ x} \langle p\ a \rangle}\ \forall L
  }{
    \dfrac{P; \forall x, p\ x \longrightarrow \langle p\ a \rangle}{P; \forall x, p\ x \longrightarrow \langle \forall x, p\ x \rangle}\ \forall R
  }\ \mathit{DECIDE}
}{P \looparrowright \forall x, p\ x}\ \text{co-fix}
$$

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\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{}{P; \forall x, p\ x \xrightarrow{p\ a} p\ a}\ \text{Initial}
        \qquad
        \dfrac{
          \dfrac{
            \dfrac{}{P; \forall x, p\ x \xrightarrow{p\ (f\ a)} p\ (f\ a)}\ \text{Initial}
          }{P; \forall x, p\ x \xrightarrow{\forall x, p\ x} p\ (f\ a)}\ \forall L
        }{P; \forall x, p\ x \longrightarrow p\ (f\ a)}\ \text{DECIDE}
      }{P; \forall x, p\ x \xrightarrow{p\ (f\ a) \supset p\ a} \langle p\ a \rangle}\ \supset L
    }{P; \forall x, p\ x \xrightarrow{\forall x, p(f(x)) \supset p\ x} \langle p\ a \rangle}\ \forall L
  }{P; \forall x, p\ x \longrightarrow \langle p\ a \rangle}\ DECIDE
}{
  \dfrac{P; \forall x, p\ x \longrightarrow \langle \forall x, p\ x \rangle}{P \looparrowright \forall x, p\ x}\ \text{co-fix}
}\ \forall R
$$

Finally, get $(p\ a)$ as a corollary. ... QUIZ!

Unlucky case: implicative coinductive invariant

Example

$\kappa_1 : \forall x, p(f\ x) \wedge q\ x \supset p\ x$
$\kappa_2 : q(a);$
$\kappa_3 : \forall x, q\ x \supset q(f\ x)$
Find invariant for:
$\underline{p\ a} \stackrel{apply}{\longrightarrow}^{\kappa_1} p(f\ a) \wedge q\ a \stackrel{apply}{\longrightarrow}^{\kappa_2} p(f\ a) \stackrel{apply}{\longrightarrow}^{\kappa_1}$
$p(f\ f\ a) \wedge q(f\ a) \stackrel{apply}{\longrightarrow}^{\kappa_3} p(f\ f\ a) \wedge q\ a \longrightarrow \ldots ?$

Unlucky case: implicative coinductive invariant

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$\kappa_1 : \forall x, p(f\ x) \wedge q\ x \supset p\ x$
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$p(f\ f\ a) \wedge q(f\ a) \xrightarrow{apply}{}^{\kappa_3} p(f\ f\ a) \wedge q\ a \longrightarrow \ldots ?$

	least fixed point	greatest fixed point
finite terms	$\{q\ a, q(f\ a),$ $q(f\ f\ a), \ldots\}$	$\{p\ a, p\ (f\ a), p(f\ f\ a), \ldots$ $q\ a, q(f\ a),\ q(f\ f\ a), \ldots\}$
finite and infinite terms	$\{q\ a, q(f\ a),$ $q(f\ f\ a), \ldots\}$	$\{p\ a, p\ (f\ a), p(f\ f\ a), \ldots$ $p(f^{\omega}), q\ a, q(f\ a),\ q(f\ f\ a),$ $\ldots q\ f^{\omega}\ \}$

Unlucky case: implicative coinductive invariant

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Example

$\kappa_1 : \forall x, p(f\ x) \wedge q\ x \supset p\ x$
$\kappa_2 : q(a);$
$\kappa_3 : \forall x, q\ x \supset q(f\ x)$

Find invariant for:

$\underline{p\ a} \xrightarrow{apply}^{\kappa_1} p(f\ a) \wedge q\ a \xrightarrow{apply}^{\kappa_2} p(f\ a) \xrightarrow{apply}^{\kappa_1}$

$p(f\ f\ a) \wedge q(f\ a) \xrightarrow{apply}^{\kappa_3} p(f\ f\ a) \wedge q\ a \longrightarrow \ldots ?$

	least fixed point	greatest fixed point
finite terms	$\{q\ a, q(f\ a), q(f\ f\ a), \ldots\}$	$\{p\ a, p\ (f\ a), p(f\ f\ a), \ldots q\ a, q(f\ a), q(f\ f\ a), \ldots\}$
finite and infinite terms	$\{q\ a, q(f\ a), q(f\ f\ a), \ldots\}$	$\{p\ a, p\ (f\ a), p(f\ f\ a), \ldots p(f^\omega), q\ a, q(f\ a), q(f\ f\ a), \ldots q\ f^\omega\}$

The only working coinductive invariant is $\forall x, q\ x \supset p\ x$,
QUIZ!!!

Final example

- $frStr = fix\ \lambda\ f\ x.scons\ x\ (f(s\ x)) = fix\ \lambda\ f\ x.[\ x,(f(s\ x))]$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P;CH \xrightarrow{from\ C\ [C,frStr(s\ C)]} from\ C\ [C, frStr(s\ C)]}\ \text{INIT} \qquad \dfrac{\dfrac{\dfrac{}{P;CH \xrightarrow{from\ (s\ C)\ (frStr(s\ C))} from(s\ C)\ (frStr(s\ C))}\ \text{INIT}}{P;CH \xrightarrow{CH} from\ (s\ C)(frStr(s\ C))}\ \forall L}{P;CH \longrightarrow from\ (s\ C)\ (frStr(s\ C))}\ \text{DECIDE}}{P;CH \xrightarrow{from\ (s\ C)\ (frStr(s\ C)) \supset from\ C\ [C,frStr(s\ C)]} from\ C\ [C, frStr(s\ C)]}\ \supset L}{P;CH \xrightarrow{\forall x\ y, from(s\ x)\ y \supset from\ x\ [x,y]} from\ C\ [C, frStr(s\ C)]}\ \forall L}{P;CH \longrightarrow from\ C\ [C, frStr(s\ C)]}\ \text{DECIDE}}{P;CH \longrightarrow from\ C(frStr\ C)}\ fix\beta\text{-conversion}}{P;CH \longrightarrow \forall x, from\ x,\ (frStr\ x)}\ \forall R}{P \longrightarrow \forall x, from\ x\ (frStr\ x)}\ \text{cofix}
$$

Final example

- $frStr = fix\ \lambda\ f\ x.scons\ x\ (f(s\ x)) = fix\ \lambda\ f\ x.[\ x, (f(s\ x))]$

$$
\dfrac{
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  \qquad
  \dfrac{
    \dfrac{
      \dfrac{}{P; CH \xrightarrow{from\ (s\ C)\ (frStr(s\ C))} from(s\ C)\ (frStr(s\ C))}\ \text{INIT}
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  }{P; CH \longrightarrow from\ (s\ C)\ (frStr(s\ C))}\ \text{DECIDE}
}{
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{
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        }{P; CH \longrightarrow from\ C\ [C, frStr(s\ C)]}\ \text{DECIDE}
      }{P; CH \longrightarrow from\ C(frStr\ C)}\ fix\beta\text{-conversion}
    }{P; CH \longrightarrow \forall x, from\ x,\ (frStr\ x)}\ \forall R
  }{P \longrightarrow \forall x, from\ x\ (frStr\ x)}\ \text{cofix}
}\ \supset L
$$

- get *from* 0 (*frStr* 0) as a corollary

QUIZ!!!

Current progress:

Analysis of coinductive properties of theories based on the language in which their coinductive invariants are expressed:

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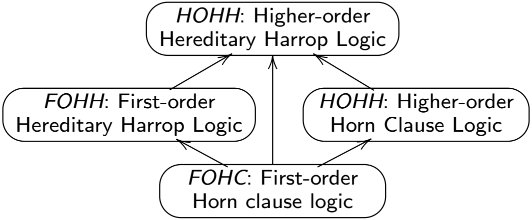
Miller and Nadathur:



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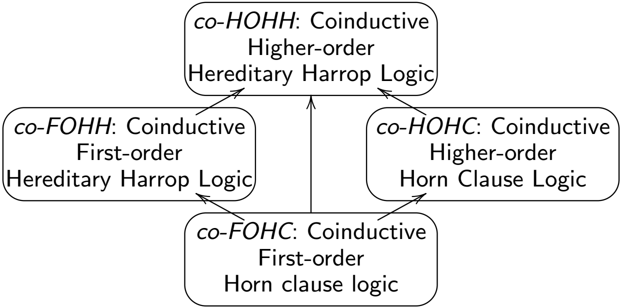
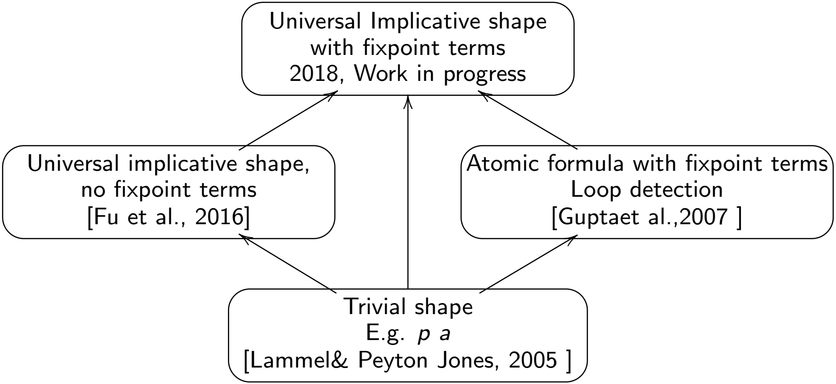
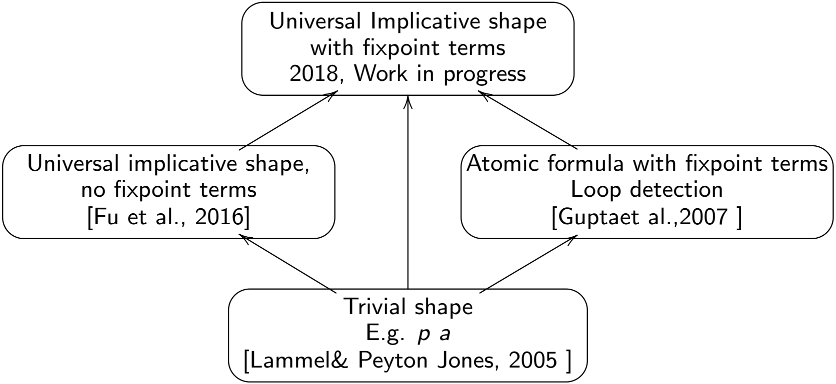


Coinductive Theory exploration



Coinductive Theory exploration



QUIZ: where CoHipster's lemmas would live?

Thanks for your attention!