



max planck institut
informatik

Extending an Isabelle Formalisation of CDCL to Optimising CDCL

Mathias Fleury

joint work with Christoph Weidenbach

Matryoshka+Veridis 2019

Let's find a model with minimal weight



	10	→		4
	20	→		13

Optimal partial model: 

Optimal total model: →  

How reliable is the theory?

Conference version

Branch and Bound for Boolean Optimization and
the Generation of Optimality Certificates

Javier Larrosa, Robert Nieuwenhuis, Albert Oliveras, and Enric Rodríguez-Carbonell (SAT 2009)

A literal l is *true* in I if $l \in I$, *false* in I if $\neg l \in I$, and *undefined* in I otherwise.

A clause set S is true in I if all its clauses are true in I . Then I is called a *model* of S , and we write $I \models S$ (and similarly if a literal or clause is true in I).

Journal version

A Framework for Certified Boolean Branch-and-Bound Optimization

Javier Larrosa, Robert Nieuwenhuis, Albert Oliveras, and Enric Rodríguez-Carbonell (JAR 2011)

literals of a clause C are false in I . A clause set S is true in I if all its clauses are true in I ; if I is also total, then I is called a *total model* of S , and we write $I \models S$.

How reliable is the theory?

amazon.de Discover Prime

Deliver to Mathias Saarbrücken 66125

Shop by Department

Mathias's Amazon Today's Deals Gift Cards Sell Help

EN Hello, Mathias Your Account Discover Prime Your Lists Shopping Basket

Amazon.de SALE Warehouse Deals Vouchers Fashion-Sale Family Students Subscribe & Save Pantry Gift Central Apps Amazon Assistant

Automation of Logic (Chapman & Hall/CRC Studies in Informatics) Hardcover – 22. July 2019

by Christoph Weidenbach (Autor)

Keine Abbildung vorhanden

Hardcover EUR 85.09

Promotion Message Vorbesteller-Preisgarantie 1 promotion

This item can be delivered to Germany - Mainland. Details

1 New from EUR 85.09

Share

EUR 85.09

Prices for items sold by Amazon are inclusive of German VAT. For other items, please see details. Vorbesteller-Preisgarantie

FREE Shipping

This title has not yet been released.

You may pre-order it now and we will deliver it to you when it arrives. Dispatched from and sold by Amazon.

Quantity: 1

Pre-order This Item Today

Pre-order now

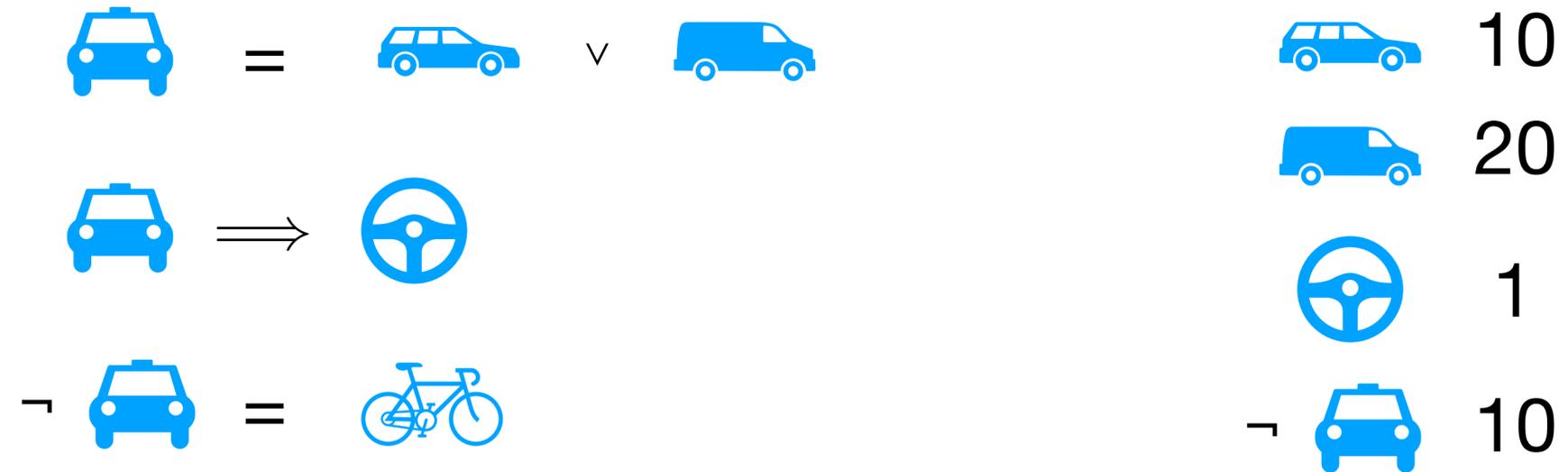
Deliver to Mathias - Saarbrücken 66125

Add to List

Report incorrect product information.

Lemma 2.15.4. The OCDCL calculus with a reasonable strategy has only 2 normal forms: $(M; N; U; 0; \perp; O)$ where $O \neq \epsilon$, $O \models N$ and $\text{cost}(O)$ is optimal, and $(M; N; U; 0; \perp; \epsilon)$ where N is unsatisfiable.

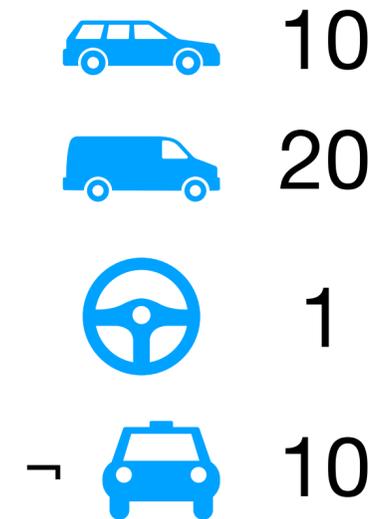
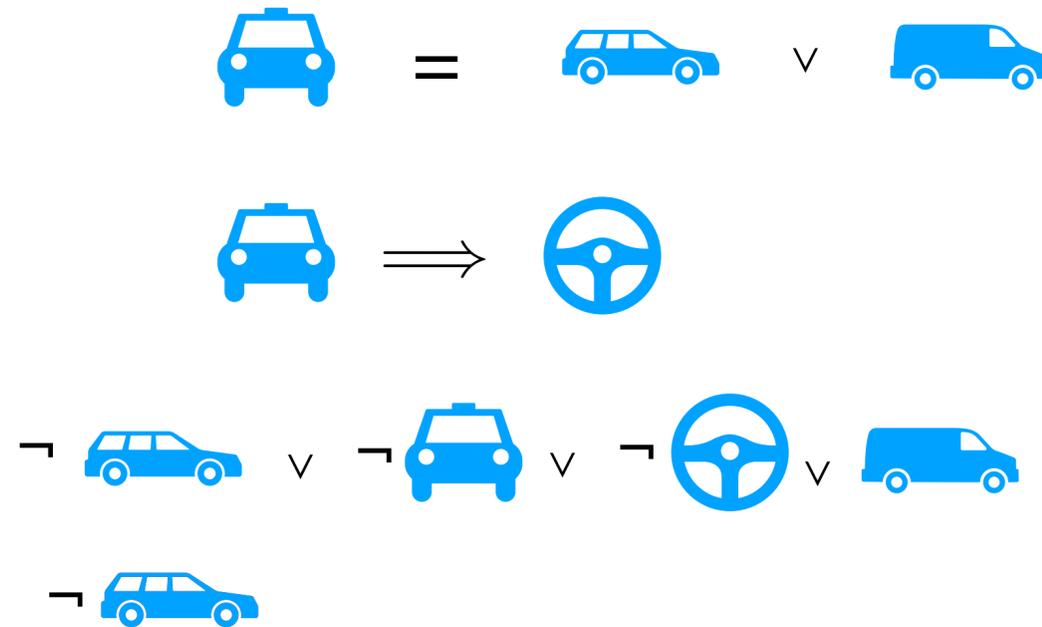
Let's optimise our problem



Let's optimise our problem

OCDCL = CDCL + identify better models
+ conflicts based on weights

Let's optimise our problem



Optimal model 11



How lazy do you like your formalisation?

Christoph's view:

$\text{OCDCL}_W = \text{CDCL} + \text{improve} + \text{conflict rules}$

copy-paste of proofs

My first idea:

$\text{OCDCL} = \text{CDCL} + \text{improve} +$
 $\{-M. \text{ cost } M \geq \text{min_cost}\}$

reuse CDCL proofs

How lazy do you like your formalisation?

My first idea:

$$\text{OCDCL} = \text{CDCL} + \text{improve} + \{-M. \text{ cost } M \geq \text{min_cost}\}$$

reuse CDCL proofs

My second idea:

$$\text{CDCL}_{\text{bnb}} = \text{CDCL} + \text{improve} + \mathcal{T}(\text{min_cost})$$

reuse CDCL proofs

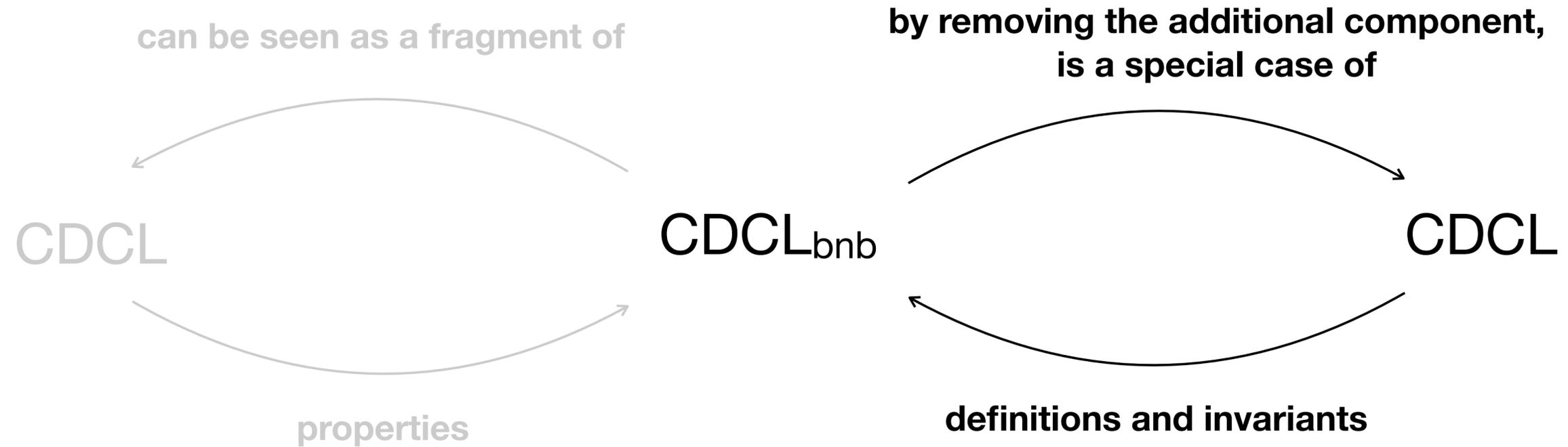
How lazy do you like your formalisation?

$$\text{CDCL}_{\text{bnb}} = \text{CDCL} + \text{improve} + \mathcal{T}(\text{min_cost})$$

$$\text{OCDCL} = \text{CDCL}_{\text{bnb}} \text{ where } \mathcal{T}(\text{min_cost}) = \{-M. \text{cost} \mid M \geq \text{min_cost}\}$$

$$\text{OCDCL}_W = \text{OCDCL} + \text{restrictions}$$

Reuse!



Reuse!

Propagate rule

in Isabelle

$$\begin{aligned} C \vee L \in N \implies M \models_{as} \neg C \implies \text{undefined_lit } M \ L \implies \\ (M, N) \Rightarrow_{\text{CDCL}} (L \# M, N) \end{aligned}$$

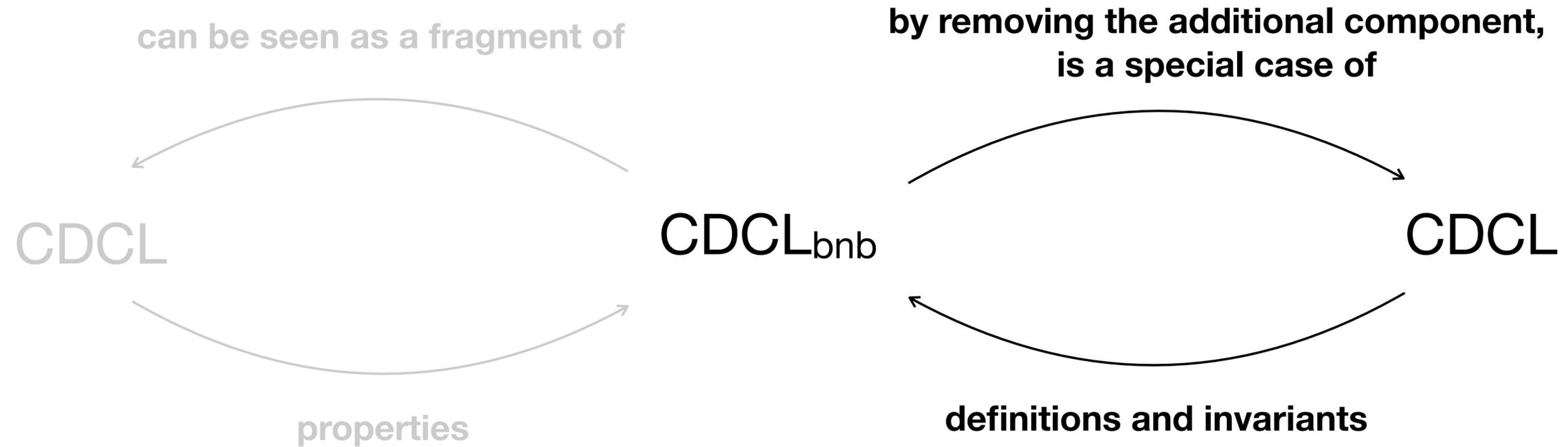
obtained for free, thanks to abstraction over the state!
also invariants and theorems can be reused

Propagate rule

in Isabelle

$$\begin{aligned} C \vee L \in N \implies M \models_{as} \neg C \implies \text{undefined_lit } M \ L \implies \\ (M, N, 0) \Rightarrow_{\text{CDCLbnb}} (L \# M, N, 0) \end{aligned}$$

Reuse!



Translate to reuse

Propagate rule

in Isabelle

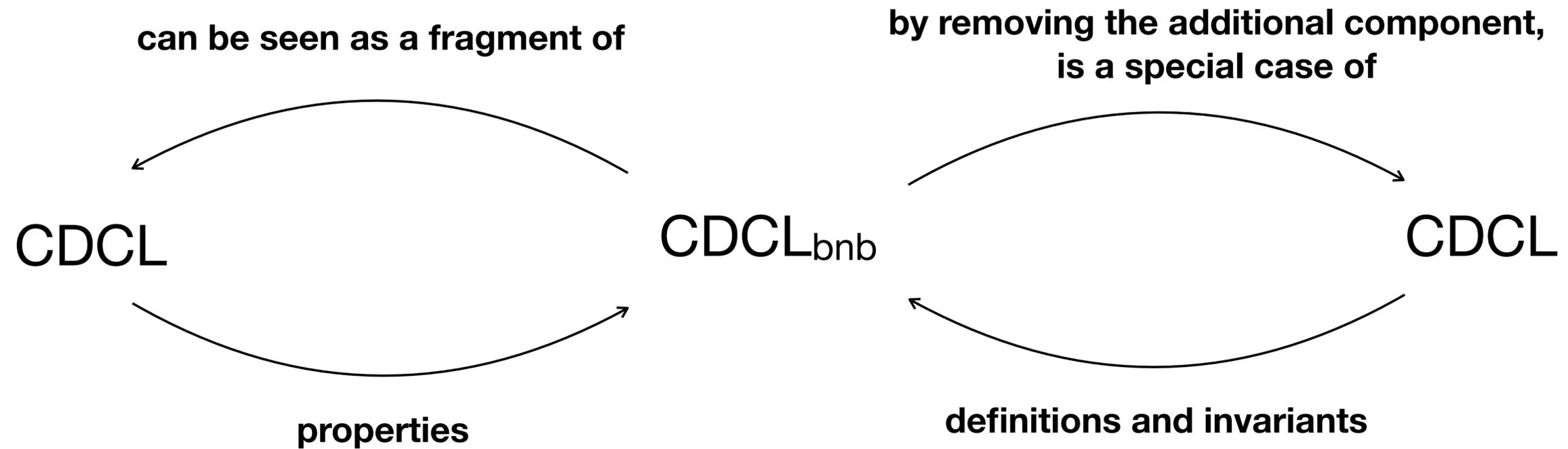
$$C \vee L \in N \implies M \models_{\text{as}} \neg C \implies \text{undefined_lit } M \ L \implies \\ (M, N, 0) \Rightarrow_{\text{CDCLbnb}} (L \# M, N, 0)$$

Propagate rule

in Isabelle

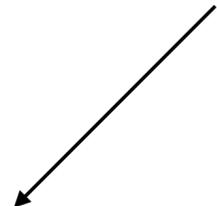
$$C \vee L \in N + \mathcal{I}(\text{min_cost}) \implies M \models_{\text{as}} \neg C \implies \\ \text{undefined_lit } M \ L \implies \\ (M, N + \mathcal{I}(\text{min_cost}), 0) \Rightarrow_{\text{CDCL}} \\ (L \# M, N + \mathcal{I}(\text{min_cost}), 0)$$

Reuse!



Reuse in practise!

ignore the additional
component



$$\text{CDCL}_{\text{bnb}} = \text{CDCL} + \text{improve} + \mathcal{I}(\text{min_cost})$$

Inherited:

Definitions (for free)

Reuse in practise!

no strategy
but terminating

well-founded
for most applications

$$\text{CDCL}_{\text{bnb}} = \text{CDCL} + \text{improve} + \mathcal{I}(\text{min_cost})$$

Inherited:

Termination (for free)

Definitions (for free)

Reuse in practise!

CDCL_{bnb} does not know anything about what is optimised!

Inherited:

Termination (for free) Definitions (for free)

Ends with an unsat set (nearly for free)

Why does it work?

OCDCL = CDCL_{bnb} where

$$\mathcal{T}(\text{min_cost}) = \{-M. \text{ cost } M \geq \text{min_cost}\}$$

Lemma

If I is a total model of N with $\text{cost} < \text{min_cost}$,
then I is a model of $N + \mathcal{T}(\text{min_cost})$

Why does it work?

Lemma

If I is a total model of N with $\text{cost} < \text{min_cost}$,
then I is a model of $N + \mathcal{T}(\text{min_cost})$

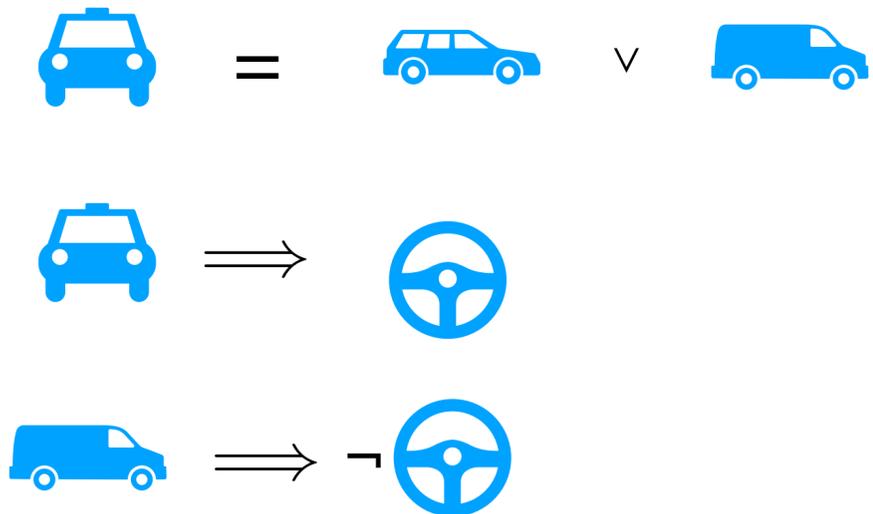
Fails for partial models!

How lazy do you like your formalisation?

make sure that the rules on paper
and in Isabelle are the same

$\text{OCDCL}_W = \text{OCDCL} + \text{restrictions}$

Another application: Dead features



Can every option be true?

How lazy do you like your formalisation?

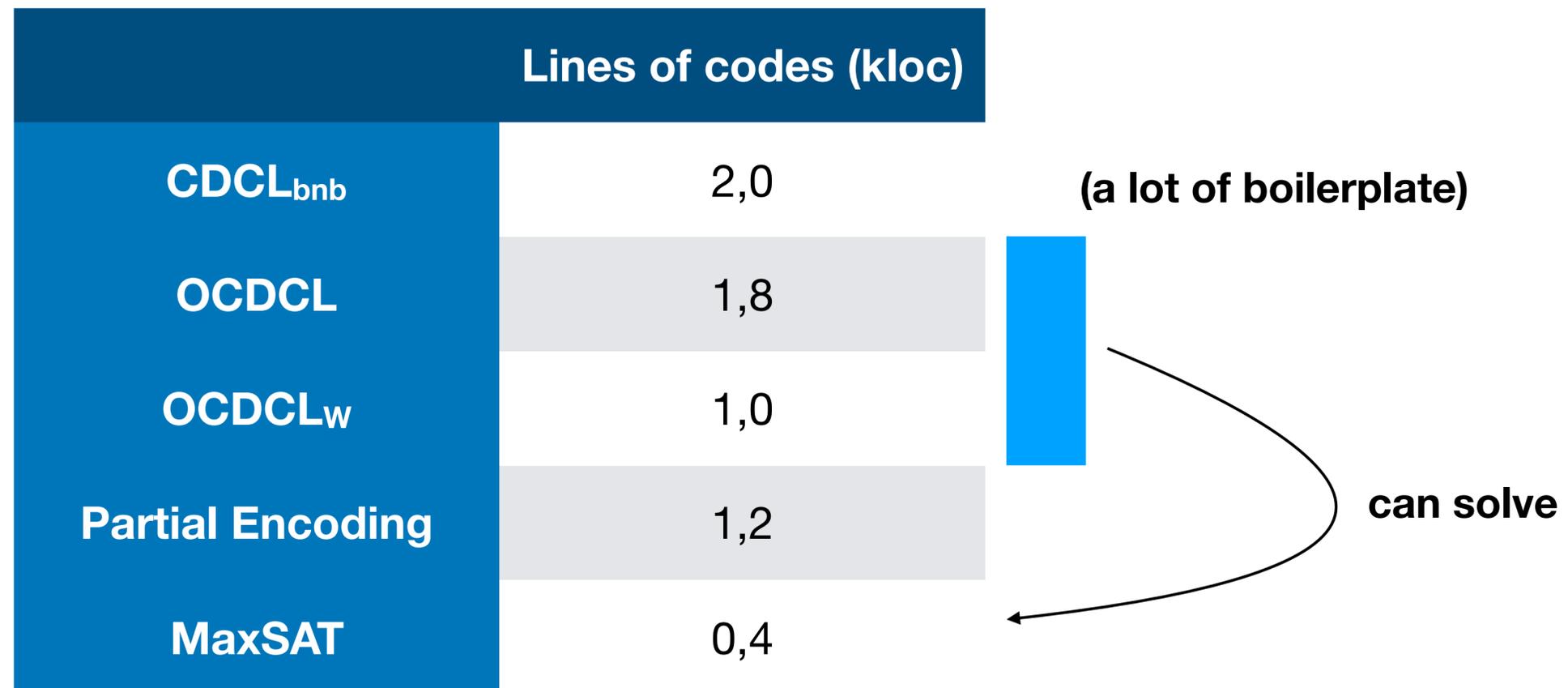
Christoph's view:

$\text{CDCLcm}_W = \text{CDCL} + \text{improve} + \text{conflict rules}$
copy-paste of proofs

My idea:

$\text{CDCLcm} = \text{CDCL}_{\text{bnb}}$ where
 $\mathcal{T}(\text{models_founds}) = \{-M. \text{ there is a model with more trues in models_founds}\}$

How lazy do you like your formalisation?



Conclusion

Concrete outcome

• CDCL with branch and bound

OCDCL = CDCL_{bnb} where

$$\mathcal{T}(\text{min_cost}) = \{-M. \text{cost } M \geq \text{min_cost}\}$$

OCDCL = CDCL_{bnb} where

$$\mathcal{T}(\text{min_cost}) = \{-D. \{M. \text{cost } M \geq \text{min_cost}\} \neq D\}$$

Future work

▶ CDCL(\mathcal{T})

Conclusion: How about CDCL(\mathcal{T})?

But isn't CDCL(\mathcal{T}) exactly:

CDCL_{bnb} where

$\mathcal{T} = \{\text{clauses entailed theory}\}$

Not exactly, because the wrong conflict clause (negation of the trail) is used

Translate to reuse

Theory propagation

Propagate rule

$$\begin{aligned} C \vee L \in N + \mathcal{T}(\text{min_cost}) &\implies M \models_{\text{as}} \neg C \implies \\ \text{undefined_lit } M \ L &\implies \\ (M, N + \mathcal{T}(\text{min_cost}), 0) &\implies_{\text{CDCLbnb}} \\ (L \# M, N + \mathcal{T}(\text{min_cost}), 0) & \end{aligned}$$

in Isabelle